Chapter Two: Integers and Variable Expressions

Name ________________________________ hour _____

Lesson One: Integers and Absolute Value
Vocabulary: opposites, integers, absolute value

Integers:

Absolute value:

Write an integer to represent each situation:
A gain of $45        a withdrawal of $75        1,500 feet below sea level

Find each absolute value.
1) | 50 |  2) | -8 |  3) | -4.5 |  4) | 5 + 6 |

Compare. Write >, <, or =.
5) | -10 | _____ | 10 |  6) 5 _____ -7  7) | -8 | _____ | -12 |
8) -12 _____ -10  9) | 65 | _____ | -150 |  10) 0 _____ | -22 |

Order the integers in each set from least to greatest.
11) -8, 6, 3, -9, 0, 15, -13
12) -20, -5, -14, 0, 25, -30

State three numbers that you could substitute for x to make the inequality true.
13) | x | ≥ 7  14) x < | -8 |  15) | x | < 4

Lesson Two: Writing and Evaluating Variable Expressions  (Part I)
Vocabulary: variable, variable expression

Variable:

Variable expression:
What verbal phrases can indicate each operation?

<table>
<thead>
<tr>
<th>operation</th>
<th>phrase</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td></td>
</tr>
<tr>
<td>-</td>
<td></td>
</tr>
<tr>
<td>×</td>
<td></td>
</tr>
<tr>
<td>÷</td>
<td></td>
</tr>
<tr>
<td>=</td>
<td></td>
</tr>
</tbody>
</table>

12 more than m

12 times y

Write the expression that goes with each of the following phrases.

16 less than y

the product of 7 and n

the quotient of r and 11

m less than 14

m less 14

the sum of 4 and 3 times x

four dollars less than Ross earned

twice as many flowers as Susan picked

nine less than the product of 3 and c is 18

12 less than the number of apples is 28

50 is two more than twice the number of miles jogged this week.

12 inches less than four times Maria's height is 228 inches.
Lesson Two, Part II
Vocabulary: Order of operations, evaluate

Order of Operations: “Please excuse my dear Aunt Sally for belching.”

\[ P = \quad\quad E = \quad\quad \]

\[ M = \quad\quad D = \quad\quad \]

(multiplication and division are worked \_______ \_______)

\[ A = \quad\quad S = \quad\quad \]

(addition and subtraction are worked \_______ \_______)

\[ FB = \quad\quad \]

Show your steps as you work through the problem.

\[ -7 + 45 \div 9 \quad 4^2 + (15 - 7) \quad 8 + 32 \div 4(-5) \quad \frac{(10)(4)}{14 - 9} \]

\[ 40 - 25 \div 5 \quad 12 + 4^2 \div 2 \quad 37 - 60 \div 3(-4) \quad \frac{19 - 12 \div 3}{24 \div 8} \]

Evaluate each expression when \( a = 2 \), \( b = 6 \) and \( c = 12 \).

\[ a - c \div b \quad -b + ac \quad bc + ab \quad ac - b^2 \]

\[ 3b + -7 \quad \frac{(2a + c)}{2} \quad -3(b - a^2) + 16 \quad \frac{36 + c}{b - a} \]
Lesson Three: Adding Integers

Vocabulary: additive inverses

When adding integers, think about positive and negative charges. A positive charge and a negative charge cancel to form a _________________.

Draw a model for adding integers using the idea of positive and negative charges.

\[-4 + 7 = \underline{\hspace{2cm}} \quad -2 + -6 = \underline{\hspace{2cm}} \quad -8 + 3 = \underline{\hspace{2cm}} \quad 6 + -7 = \underline{\hspace{2cm}}\]

Write a guideline for adding two integers when the signs are alike.

Write a guideline for adding two integers when the signs are different.

Solve.

\[-45 + -5 = \underline{\hspace{2cm}} \quad -9 + 3 = \underline{\hspace{2cm}} \quad -32 + 40 = \underline{\hspace{2cm}}\]

\[12 + -12 = \underline{\hspace{2cm}} \quad -17 + -9 = \underline{\hspace{2cm}} \quad 9 + 20 = \underline{\hspace{2cm}}\]

\[-10 + 6 = \underline{\hspace{2cm}} \quad 9 + -7 = \underline{\hspace{2cm}} \quad -7 + -40 = \underline{\hspace{2cm}}\]

\[9 + (-8) = \underline{\hspace{2cm}} \quad -20 + 30 = \underline{\hspace{2cm}} \quad -18 + (-12) = \underline{\hspace{2cm}}\]

\[-32 + 10 = \underline{\hspace{2cm}} \quad 22 + -23 = \underline{\hspace{2cm}} \quad -15 + -15 = \underline{\hspace{2cm}}\]

\[-12 + -3 + 18 = \underline{\hspace{2cm}} \quad -30 + 15 + -5 = \underline{\hspace{2cm}}\]
Lesson Four: Subtracting Integers

The sign “-” can be read as ____________________________ or ____________________________.

Method ONE:

4 - 8 = ___

Take the signs ____________________________

We have two numbers, ___________ and ___________

Add those numbers. The sum is ___________

-12 - -4

-4 means ____________________________

The two numbers we add together are ___________ and ___________

The sum is ___________

Method TWO:

6 - 8

Change the subtraction sign to an ____________________________ sign

Change the sign of the second number.

The new problem would be ____________________________ = _______

-9 - -3

Change the subtraction sign to an ____________________________ sign

Change the sign of the second number.

The new problem would be ____________________________ = _______

Examples:

- 14 - -2 = _____ 17 - 7 = _____ -9 - 9 = _____

-5 - (-5) = _____ 15 - -5 = _____ -8 - -3 = _____

8 - 12 = _____ -20 - -5 = _____ -13 - (-3) = _____

30 - -5 = _____ -24 - 6 = _____ -18 - 2 = _____

40 - 50 = _____ -11 - -1 = _____ 15 - -2 = _____

-10 - -3 - -2 = _____ 15 - 20 - -5 = _____

-24 - 6 - -10 = _____ 3 - -7 - 8 = _____
Write the next four numbers in the pattern. Explain the pattern.

40, 36, 32, 28, 24, _____, _____, _____, _____

-28, -18, -11, -4, _____, _____, _____, _____

Lesson Five: Multiplying and Dividing Integers

A shortcut for multiplying and dividing integers: Count the number of negatives.

→ an even number of negatives:

→ an odd number of negatives:

Will the answers to the following problems be positive or negative?

\((-3)(-4)(12) = \) _____ \((-1)(-2)(-3)(-4) = \) _____ \((+)(-)(-)(+)(-) = \) _____

Solve the following:

\(6(-5) = \) _____ \(-8(-10) = \) _____ \((4)(-3) = \) _____

\(2(-11) = \) _____ \(-2(3)(-5) = \) _____ \((-15)(-2) = \) _____

\(-\frac{9}{3} = \) _____ \(\frac{14}{-7} = \) _____ \(-\frac{90}{-10} = \) _____

\(-\frac{45}{9} = \) _____ \(-\frac{36}{-6} = \) _____ \(-\frac{25}{-5} = \) _____

Lesson Six: Exponents and Multiplication

Vocabulary: base, exponent, power

In the expression \(6^3\), six is the _______ and three is the _______.

The problem would be solved by multiplying. (______)(______)(______) = _____
\((-7)^4 = \)  
\(-5^2 = \)

If we were to write \((4)(4)(4)(4)(4)\) in \(\) form, it would be \(\)

Write the following in exponential form. (Write using a single exponent.)

\((5)(5)(5)(5)(5) = \)  
\((-n)(-n)(-n)(-n) = \)

\((12)(x)(x)(x) = \)  
\((-4y)(-4y)(-4y) = \)

\(-(3a)(3a)(3a)(3a) = \)  
\(-(3a)(-3a)(-3a)(3a) = \)

Evaluate each expression for the given value.

\(3a^4\) for \(a = 2\)  
\((3a)^4\) for \(a = 2\)  
\(-4x^2\) for \(x = 5\)

\((-4x)^2\) for \(x = 5\)  
\(5y^3\) for \(y = 3\)  
\((5y)^3\) for \(y = 3\)

Multiplying Powers with the Same Base: to multiply numbers or variables with the same base, 

Arithmetic example:

Algebraic example:

Write each expression using a single exponent:

\((4^5)(4^2) = \)  
\((n^4)(n^6) = \)  
\((12^3)(12^4)(12^5) = \)

\((-11)^3(-11)^4 = \)  
\((n)(n^4)(n^2) = \)  
\((-x^4)(-x^5) = \)
Lesson Seven: Evaluating expressions with exponents

Evaluate each expression:

\((-5)^2 + 6 \cdot 3\) = ______  \[10 \cdot 4 - 2^3 = _____ \quad 8 + (11 - 5)^2 = _____\]

\[4(5^3) + 12 \div 4 = ____ \quad 2^3 \cdot 4 - 6^2 \div 4 = _____ \quad -12 + 3^2 (15 - 2^3) = ____\]

Evaluate each expression for the given value of the variable.

\[8y^3; \ x = 2 \quad -3m^2; \ m = 6 \quad 3n^2 + 4n + 2; \ n = 5\]

\[-7x^3 \div x^2; \ x = -4 \quad 5m^4; \ m = -2 \quad 8x^2y^3; \ x = 3, \ y = 3\]

Evaluate each expression for \(x = -2.4, \ y = -3, \ z = 5\)

\[y^3 + 12 \quad x^2 - z^3 \quad 15xy^3\]

\[x^2y^2z^2 \quad (z - y)^2 \quad z^3 \div -z\]
Lesson Ten: Exponents and Division

Dividing powers with the same base: To divide numbers or variables with the same base,

- Arithmetic example:

- Algebraic example:

Zero as an exponent: for any nonzero number \( a \), \( a^0 = \)_____

Negative exponent: For any nonzero number \( a \) and any integer \( n \), \( a^{-n} = \)_____

Simplify each expression:

\[
\frac{m^5}{m^3} \quad \frac{y^{11}}{y} \quad \frac{s^{10}}{s^4} \quad \frac{n^{15}}{n^5}
\]

\[
\frac{c^{12}}{c^2} \quad \frac{a^4}{a} \quad \frac{6a^5}{3a^2} \quad \frac{15m^4}{5m}
\]

\[
\frac{18n^{10}}{-2n^7} \quad \frac{-5m^7}{25m^3} \quad \frac{12x^2y^4}{6xy} \quad \frac{a^2b^3c^4}{ab^2c^3}
\]

Write each expression as an integer or simple fraction.

\[
(-4)^{-3} \quad -4^{-2} \quad \frac{5^0}{5^2} \quad \frac{1}{3^{-2}}
\]

\[
\frac{-3^5}{(-3)^5} \quad \frac{(-3)^2}{(-3)^4} \quad (-6)^{-2} \quad \frac{6^0}{6^4}
\]
Simplify each expression. Use only positive exponents.

\[ \frac{m^{-3}}{m^{-8}} \quad \frac{m^{5}}{m^{7}} \quad \frac{a^{3}}{a^{7}} \quad \frac{3^{-2}y^{3}}{2x^{-2}} \]

Complete each equation.

\[ \frac{4^{12}}{4^{\Box}} = 4^{5} \quad \frac{x^{7}}{x^{\Box}} = x^{2} \quad \frac{12a^{10}}{6a^{4}} = 2a^{\Box} \]

\[ \frac{1}{3^{4}} = 3^{\Box} \quad \frac{1}{5^{8}} = 5^{\Box} \quad \frac{1}{(-3)^{8}} = (-3)^{\Box} \]

\[ \frac{1}{m^{12}} = m^{\Box} \quad \frac{m^{15}}{m^{\Box}} = m^{-5} \quad \frac{y^{\Box}}{y^{12}} = y^{-2} \]